Engineering Notes

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Effect of Downwash on the Induced Drag of Canard-Wing Combinations

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Nomenclature

b = span
B_n = Fourier coefficient in expansion of W_T
C_n = Fourier coefficient in expansion of K_2
D_i = induced drag
g = biplane gap
K = circulation
K_E = elliptic circulation
K_I = induced circulation
L = lift
q = dynamic pressure
R = aspect ratio
T_i = induced thrust
V = freestream velocity
W = total downwash
W_E = downwash due to elliptic circulation
W_I = downwash due to induced circulation
W_T = Trefftz-plane downwash of surface 1
y = spanwise coordinate
α = angle of incidence
$\bar{\alpha}_2$ = geometrical incidence of surface 2
θ = Fourier expansion parameter = $\cos^{-1}(-2y_2/b_2)$
μ = lift curve slope parameter = $2\sin\theta/R$
ν = induced thrust coefficient
ρ = density
σ = Prandtl coefficient

Subscripts

1 = forward surface
2 = rearward surface
opt = optimum value

I. Introduction

RECENTLY, a number of papers[1-5] have appeared concerning the induced drag of canard-wing and wing-tail aircraft, based on Prandtl's biplane theory. Prandtl calculated the mutually induced drag of a biplane by assuming that the spanwise lift distribution on each surface was elliptical and integrating the product of the downwash and lift across the span. Application to canard-wing and wing-tail configurations is achieved by invoking Munk's stagger theorem, viz., the total induced drag of a multiplane system is unaltered if any of the lifting surfaces is moved parallel to the direction of motion, provided that the element is adjusted in attitude to maintain *the same spanwise distribution* of lift on each surface. Thus, the theory rests on the assumption that the lift distribution on each surface is elliptical *under the influence of the downwash field of the other,* and it is clear that, in order to satisfy such a condition, a lift-dependent twist needs to be imparted to one or both surfaces.

In this Note, the induced drag of canard-wing and wing-tail combinations is calculated for the limiting case in which the downstream surface is located in the Trefftz plane (infinite stagger), but based on the assumption that the loading on each surface is elliptical *in isolation.* It is shown that additional terms associated with induced circulation effects act to reduce the drag over that calculated using the classical theory. This induced thrust component is shown to be very small for the wing-tail configuration, but significant for the canard-wing layout. In addition, given elliptic loading on the upstream surface, an expression is derived for the loading distribution on the downstream surface which is optimum in the sense that the induced thrust component is maximized. The Trefftz plane assumption leads to a simplification of the analysis, since it means that the lift distribution of the downstream surface does not influence the drag of the upstream surface. The case of finite stagger, including the effects of bound vorticity on both surfaces, will be covered in a subsequent paper.

II. Analysis

We consider the problem of the induced drag of an elliptically loaded surface (subscript *1*) and a downstream lifting surface (subscript *2*) located in the Trefftz plane. Surface 2 has an elliptical loading distribution in isolation, but the loading is nonelliptical under the influence of the downwash field of 1. As shown by Laitone,[3] the upstream surface is unaffected by the lift on surface 2 and the self-induced drag of 1 is given by $D_{i1} = L_1^2/(\pi q b_1^2)$. The induced drag acting on 2 can be expressed in the form

$$D_{i2} = \int_{-b_2/2}^{b_2/2} \rho W(y_2,g) K_2(y_2,g)\,\mathrm{d}y_2 \tag{1}$$

where $W(y_2,g)$ is the total downwash, $K_2(y_2,g)$ is the circulation, g is the biplane gap, and y_2 is the spanwise coordinate. The circulation K_2 may be expressed in terms of a Fourier series,[6] so that

$$K_2 = \frac{2V}{\pi q}\left(\frac{L_2}{b_2}\sin\theta + \frac{L_1}{b_1}\sum_{n=3}^{\infty} C_n \sin n\theta\right) = K_{E2} + K_{I2} \tag{2}$$

where $\cos\theta = -2y_2/b_2$, $K_{E2} = 2L_2 V\sin\theta/\pi q b_2$ is the elliptic circulation, and K_{I2} is the induced circulation. It should be noted that, since the loading is symmetrical, the even coefficients in the Fourier expansion are zero. The downwash associated with K_2 is then given by

$$W_2 = \frac{V}{\pi q}\left(\frac{L_2}{b_2^2} + \frac{L_1}{b_2 b_1}\sum_{n=3}^{\infty}\frac{nC_n \sin n\theta}{\sin\theta}\right) = W_{E2} + W_{I2} \tag{3}$$

where $W_{E2} = VL_2/\pi q b_2^2$ is the (constant) downwash from the elliptic circulation and W_{I2} is the downwash from the induced circulation. The downwash W_T due to the forward surface 1 is determined from the exact solution for the Trefftz-plane downwash of an elliptically loaded line.[7] To maintain compatibility with the Fourier series representations above,

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this component is expressed in the nonstandard form

$$W_T = \frac{2L_1 V}{\pi q b_1 b_2} \sum_{n=1}^{\infty} \frac{B_n \sin n\theta}{\sin\theta} \tag{4}$$

The calculation of the coefficients B_n is discussed in Sec. III. From Eqs. (1-4), the induced drag on the downstream surface is given by

$$D_{i2} = \int_0^{\pi} \frac{\rho b_2}{2} (K_{E2} + K_{I2})(W_{E2} + W_{I2} + W_T) \sin\theta d\theta$$

$$= \frac{1}{\pi q} \left[\frac{L_2^2}{b_2^2} + 2B_1 \frac{L_1 L_2}{b_1 b_2} + \frac{L_1^2}{b_1^2} \sum_{n=3}^{\infty} C_n (2B_n + nC_n) \right] \tag{5}$$

Here, the first term is the self-induced drag of 2, the second term is the mutually induced drag, with the Prandtl coefficient $\sigma = B_1$. The third term arises from the induced circulation and is not included in the classical theory.

The Fourier coefficients C_n and B_n can be related via the fundamental monoplane equation for lift in terms of angle of incidence (see Durand,[7] p. 172)

$$\mu\alpha\sin\theta = \frac{L_2}{\pi q b_2^2} \sin\theta(\mu + \sin\theta)$$

$$+ \frac{L_1}{\pi q b_1 b_2} \sum_{n=3}^{\infty} C_n \sin n\theta (n\mu + \sin\theta) \tag{6}$$

If surface 2 has an elliptical planform, $\mu = 2\sin\theta / R$, where R is the aspect ratio. Further, since the loading in isolation is assumed to be elliptical, the surface must be untwisted and hence the sectional angle of incidence is composed of a constant geometrical component $\bar{\alpha}_2$ and a downwash term, so that $\alpha = \bar{\alpha}_2 - W_T / V$. Then, inserting these relationships into Eq. (6) and using Eq. (4) we have $C_n = -4B_n / (2n + R)$. Thus the final term in Eq. (5) can be expressed as an induced thrust, T_{i2}, defined as a proportion of the self-induced drag of surface 1 by $T_{i2} = \nu D_{i1}$, where ν is the induced thrust coefficient given by

$$\nu = 8R \sum_{n=3}^{\infty} \frac{B_n^2}{(2n+R)^2} \tag{7}$$

Hence, the inclusion of induced circulation effects in the analysis yields a reduction in the total induced drag over that predicted using classical biplane theory and the total induced drag is then given by

$$D_i = \frac{1}{\pi q} \left((1-\nu) \frac{L_1^2}{b_1^2} + 2\sigma \frac{L_1 L_2}{b_1 b_2} + \frac{L_2^2}{b_2^2} \right)$$

Finally, we consider the circulation distribution on surface 2, which yields the maximum value of the induced thrust. Here, rather than imposing the condition that the downstream surface has an elliptic loading in isolation and using Eq. (6) to relate the coefficients C_n and B_n, we can determine the sequence C_n $(n = 3,5,7,...)$ which optimizes the induced thrust. It can be shown that $C_{n_{\rm opt}} = -B_n / n$, and that the optimum induced thrust coefficient is given by

$$\nu_{\rm opt} = \sum_{n=3}^{\infty} \frac{B_n^2}{n} \tag{8}$$

The relationship between ν and $\nu_{\rm opt}$, defined by Eqs. (7) and (8), is one of the items discussed in the following section.

III. Results

The Trefftz-plane downwash W_T of the upstream surface 1 can be deduced from the exact solution for an elliptically loaded line given by Durand,[7] (p. 148). The Fourier coefficients, B_n, are determined by minimizing the mean square error between a set of spanwise values of W_T taken from the exact solution and the series representation Eq. (4). In general, 100 spanwise stations and 20 terms in the Fourier expansion (i.e., $B_1, B_3, ..., B_{39}$) have been found sufficiently accurate. It should be noted that the exact solution for the downwash W_T given by Durand,[7] becomes singular as the gap between the surfaces, $g \to 0$ and $y \to \pm b_1/2$. In reality, viscous effects will act to eliminate the singularity, but the inclusion of such effects is outside the scope of the current investigation.

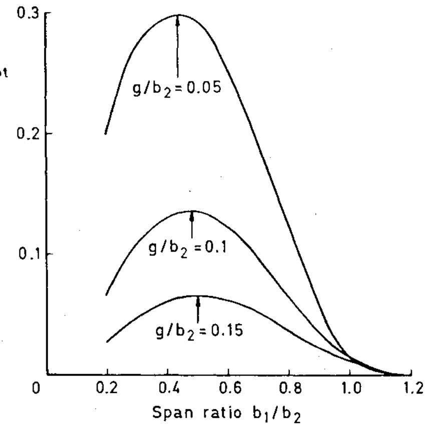

Fig. 1 Effect of span ratio b_1/b_2 and gap ratio g/b_2 on optimum induced thrust coefficient $\nu_{\rm opt}$.

From Eq. (7), the effect of aspect ratio R on the induced thrust coefficient ν can be calculated and compared with the optimum value $\nu_{\rm opt}$, given by Eq. (8). It is found that, in general, for values of R between 4 and 10, ν lies within 5% of $\nu_{\rm opt}$ and hence the optimum value can be taken to be a sufficiently accurate representation of the induced thrust for a rearward surface with elliptical planform and zero twist. In view of the association of elliptic loading with minimum induced drag conditions for the monoplane, it is interesting to note that a similar close relationship obtains for the biplane with infinite stagger.

Values of $\nu_{\rm opt}$ as a function of b_1/b_2 and g/b_2 are illustrated in Fig. 1. It can be seen that the induced thrust coefficient reaches a maximum for a span ratio, b_1/b_2 of approximately 0.5 and falls rapidly towards zero for $b_1/b_2 > 1$. Hence the correction is negligible for the wing-tail configuration with $b_1 > b_2$, and is small, but significant, for the canard wing with $b_1 < b_2$. For example if $b_1/b_2 = 0.5$, $g/b_2 = 0.05$, $L_1 = 0.25$, $L_2 = 0.75$, the induced thrust contribution amounts to some 6% of the total induced drag calculated using the Prandtl theory.

References

[1] Laitone, E.V., "Ideal Tail Load for Minimum Aircraft Drag," *Journal of Aircraft*, Vol. 15, March 1978, pp. 190-192.

[2] Laitone, E.V., "Positive Tail Loads for Minimum Induced Drag of Subsonic Aircraft," *Journal of Aircraft*, Vol. 15, Dec. 1978, pp. 837-842.

[3] Laitone, E.V., "Prandtl's Biplane Theory Applied to Canard and Tandem Aircraft," *Journal of Aircraft*, Vol. 17, April 1980, pp. 233-237.

[4] Wolkovitch, J., "Subsonic VSTOL Aircraft Configurations with Tandem Wings," *Journal of Aircraft*, Vol. 16, Sept. 1979, pp. 605-611.

[5] Butler, G.F., "An Analytical Study of the Induced Drag of Canard-Wing-Tail Aircraft Configurations with Various Levels of Static Stability," RAE Technical Report 81052, 1981.

[6] Glauert, H., "The Elements of Aerofoil and Airscrew Theory," University Press, Cambridge, 1926.

[7] Durand, W.F., *Aerodynamic Theory*, Vol. II, Division E., Springer, Berlin, 1935.